Rigid body simulations and related research

Habilitation thesis

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Abstract

In this habilitation thesis we have described the professional activity conducted by the candidate after obtaining his PhD degree from University of Maryland, Baltimore County, in 2006. The research results presented here are concerned with the simulation of rigid body systems with contact and friction and with mathematical inequalities with applications to probability and statistics.

The thesis is structured in three chapters. In the first chapter, we introduce the terminology and present introductory material related to the simulation of rigid body systems with joints, contacts and Coulomb friction. In the absence of contacts, the rigid body system is modeled by a system of differential algebraic equations (DAEs). The rigid body DAE is an index 3 DAE. The numerical schemes presented here use an equivalent index 2 reformulation.

When frictional contacts are present, unilateral and complementarity constraints become part of the mathematical model giving rise to a *differential complementarity problem (DCP)*. It has been shown, that even when no impacts occur the DCP is not compatible with the Coulomb friction model. This is due to the fact that impulsive forces can act even when no impact occurs and it was pointed out by P. Painlevé in 1895. Therefore a more appropriate continuous-time formulation for rigid body systems experiencing collisions and frictional contacts is given by *measure differential inclusions (MDIs)*. One of the key elements in the MDI formulation is the *friction cone*.

The numerical integration schemes (time-stepping schemes) presented here formulate the integration step as a *mixed linear complementarity problem (MLCP)*. The MLCP is a mixture of a *linear complementarity problem (LCP)* with a system of linear equations. When Coulomb friction is part of the mathematical model the matrix of the underlying LCP is copositive.

In the second chapter of this thesis the main contributions are presented. We start with the contributions related to the simulation of rigid body systems and end with auxiliary results consisting of mathematical inequalities with applications to probability and statistics. First, a class of linearly implicit time-stepping schemes, [7], was presented here. The schemes solve a mixed linear complementarity problem (MLCP) at each integration step. To obtain the MLCP formulation, the full (nonlinear) frictional cone is replaced by a polyhedral approximation. We

addressed the issue of convergence of these time-stepping schemes in the context of measure differential inclusions. One of the key assumptions in proving the convergence results is given by the *uniform pointedness of the total friction cone*. The analysis performed in [7] introduces two new concepts the *reduced friction cone* and the corresponding *reduced measure differential inclusion*.

We continue by describing the results of [8], where a quadratic programming (QP) based model for granular flow simulation inside of a pebble bed reactor was analyzed from a computational point of view. The integration step is formulated as a QP, which is obtained via convex relaxation. Both the primal and the dual formulations are considered. The dual program is a bound-constrained QP, which allows for the use of some bound constrained minimization solvers. Again the pointedness of the friction cone plays an important role and it is shown that together with the feasibility of the primal QP guarantees no duality gap.

Next we described the results obtained in [2], [1] and [3], where rigid body systems in a quasi-static setting were analyzed. In such a framework, Newton' s second law which is specific to a dynamical setting is replaced by an equilibrium equation. The application described in [2] and [1] is known as the peg-in-the-hole problem at the meso-scale. A planar rigid part (peg) is manipulated by pushing operations and the task is to take the peg from an initial sensed configuration A to the goal configuration B. The quasi-static model is motivated by the fact that inertial forces are one order smaller than the frictional forces. There are uncertainties due to sensing, modelling and parameter estimation. Since uncertainties play an important role in such a system, designing controls that will prove robust in the presence of uncertainties is extremely important. In [2] and [1], robust manipulation primitives were designed mostly based on geometric considerations. The robust controls can be used then in the context of randomized planning. Here, we have also shown how the analysis of the LCP structure may lead to the selection of robust controls. More precisely, one can use complementarity matrices to decide whether the applied controls are robust or not. A change of the complementarity matrix will be equivalent to a switching event and the corresponding control will be rejected.

An optimization problem appearing in optical flow estimation is analyzed next. The l_1 minimization problem from optical flow was presented in [6]. The optimization problem is obtained from the discrete version of the classical Horn-Schunck model. The standard l_2 energy functional is replaced by its l_1 counterpart. For the l_1 minimization problem, two linear programming reformulations were analyzed in [6]. Here we have presented the lines of our analysis for the LP with a better structure from the matrix sparsity point of view. Primal dual interior point methods (IPM) can be used to solve this LP and the sparse structure specific to this optical flow problem can be exploited in the context of parallel algorithms.

The last part of the second chapter is concerned with some mathematical inequalities with applications to probability and statistics. In [4], a Chebysev–Grüss type inequality was given. The results developed here use the modulus of continuity and its least concave majorant, for the case of two linear positive functionals which preserve the constants. The new results of [4] can be used in various probabilistic applications. We have described here how these results can be used in the estimation of covariances for different pairs of random variables. In [5], several new inequalities of the Hermite-Hadamard type were obtained. Here, we presented one such result, that can be immediately used in the estimation of moments of continuous random variables.

In the third and last chapter we present some of the lines that characterizes the present and future research work. Some of the research items which are part of our current and future research are given below.

- Design and implementation of LCP based time-stepping schemes for autonomous navigation. This can be done by allowing virtual contacts. In this context, we plan in using the underlying LCP structure of the integration step in deciding whether switching events occur or not.

- Design of randomized algorithms for solving systems of nonlinear equation.

- Theoretical and computational results related to a class of stochastic optimization problems with mixed expectation and per-scenario constraints (abbreviated by SOESC). From an analytical point of view, we are interested in convergence results for sample average approximations applied to SOESC problems. From a computational point of view, our interest resides in designing parallel algorithms that would exploit the particular structure of these problems in the context of interior point methods.

- Design and analysis of new time-stepping schemes for both index 2 and index 3 rigid body DAEs.

- Convergence results in the measure differential inclusion sense for systems experiencing partially elastic collisions.

Bibliography

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